Summary of Convergence Tests

NAME	STATEMENT	COMMENTS
Divergence Test	If $\lim_{n\to\infty} a_n \neq 0$, then $\sum a_n$ diverges.	If $\lim_{n\to\infty} a_n = 0$, then $\sum a_n$ may or may not converge.
P-Test	Let $\sum \frac{1}{n^p}$ be a series with positive terms, then (a) Series converges if $p>1$ (b) Series diverges if $p\leq 1$	This test can be used in conjunction with the comparison test for any a_n whose denominator is raised to the n^{th} power.
Comparison Test	Let $\sum a_n$ and $\sum b_n$ be series with non-negative terms such that $a_1\leq b_1$, $a_2\leq b_2$, $a_3\leq b_3$, … if $\sum b_n$ converges, then $\sum a_n$ converges, and if $\sum a_n$ diverges, then $\sum b_n$ diverges.	Try this test as a last resort since other tests are often easier to apply.
Limit Comparison Test	Let $\sum a_n$ and $\sum b_n$ be series with positive terms such that $\ell = \lim_{n\to\infty} \frac{a_n}{b_n}$ if $0 < \ell < \infty$, then both series converge, or both series diverge.	This is easier to apply than the Comparison Test, but requires some intuition in choosing the appropriate series $\sum b_n$ for comparison.
Integral Test	Let $\sum a_n$ be a series with positive terms, and let f(x) be the function that results when n is replaced by x in the n^{th} term of the associated sequence. If f(x) is decreasing and continuous for $x \geq 1$, then $\sum_{n=1}^{\infty} a_n$ and $\int_1^{\infty} f(x)dx$ both converge or both diverge.	This test only works for series that have positive terms. Try this test when f(x) is easy to integrate.
Ratio Test	Let $\sum a_n$ be a series with positive terms and suppose $\ell = \lim_{n\to\infty} \frac{a_{n+1}}{a_n}$ (a) Series converges if $\ell < 1$ (b) Series diverges if $\ell > 1$ (c) Test fails if $\ell = 1$	Try this test when a_n involves factorials or n^{th} powers.
Ratio Test for Absolute Convergence	Let $\sum a_n$ be a series and suppose $\ell = \lim_{n\to\infty} \left\lvert\frac{a_{n+1}}{a_n}\right\rvert$ (a) Series converges if $\ell < 1$ (b) Series diverges if $\ell > 1$ (c) Test fails if $\ell = 1$	This is the default test because it is one of the easiest tests and it rarely fails. Note: the series need not have only positive terms nor does it have to be alternating.
Root Test	Let $\sum a_n$ be a series and suppose $\ell = \lim_{n\to\infty} \sqrt[n]{\lvert a_n\rvert}$ (a) Series converges if $\ell < 1$ (b) Series diverges if $\ell > 1$ (c) Test fails if $\ell = 1$	This test is the most accurate, but not the easiest to use in many situations. Use this test when a_n has n^{th} powers.
Alternating Series Test	If $a_n > 0$ for all n, then the series $a_1 - a_2 + a_3 - a_4 + \ldots$ or $-a_1 + a_2 - a_3 + a_4 - \ldots$ Converge if the following conditions hold: (a) $a_1 > a_2 > a_3 > a_4 > \ldots$ (b) $\lim_{n\to\infty} a_n = 0$	This test <u>only</u> applies to alternating series.